Surrogate Modeling of Buckling Analysis in Support of Composite Structure Optimization

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Abstract:

Problem of aircraft structural components (wing, fuselage, tail) optimization is considered. Solution of this problem is very computationally intensive, since it requires at each iteration a two-level process: first from previous iteration, an update step at full component level must be performed in order to take into account internal loads and their sensitivities in the whole structure involved by changes in local geometry. Second, numerous local analyzes are run on isolated elements (for example, super stiffeners) of structural components in order to calculate mechanical strength criteria and their sensitivities, depending on current internal loads. An optimization step is then performed from combined global-local sensitivities. This bi-level global-local optimization process is then repeated until convergence of load distribution in the whole structure. Numerous calculations of mechanical strength criteria are necessary for local analyzes, resulting in great increase of the time between two iterations. In this work an effective method for speeding up the optimization process was proposed. The method uses surrogate models of optimization constraints (mechanical strength criteria) and provides a reduction of structure optimization computational time from several days to a few hours.

***Key Words*: Composite Structure, Surrogate Modeling, Optimization.**

1 Introduction

Aeronautical structures are mainly made of stiffened panels, i.e. thin shells (also called skin) enforced with stiffeners (respectively called frames and stringers) in both orbital and longitudinal directions. For the sake of study the whole structure is divided into elementary parts called Super Stiffeners, consisting in the theoretical union of a stringer and two half panels. These basic structures are subject to highly non-linear phenomena such as buckling, collapse and damage tolerance.

In order to determine the optimal size of these super stiffeners, static mechanical criteria must be computed using dedicated software that is based on non-linear calculation. Thus, the analysis and the dimension estimation of the whole structure is currently computed by running a two-level study: at

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a global level, a Finite Element (FE) analysis run on the whole FE model provides internal loads, applied to each S-Stiffener; at a local level these loads are used to compute static mechanical criteria. Most of these criteria are formulated using Reserve Factors (RF): a structure is validated provided all its RFs are greater than one.

Therefore detailed design of an aircraft fuselage requires a two-level loop: first, changes from local geometry defined at previous iteration involve a new internal load distribution in the whole structure, an update step must then be performed to take these changes into account and to compute sensitivities. Second, numerous local analyzes are run on isolated super stiffeners to compute mechanical criteria and their sensitivities, depending on current internal loads. This bi-level global-local optimization process is then repeated until convergence of load distribution in the whole structure.

Local mechanical criteria are computed by local methods, which are used because of huge dimensionality of the problem (O(10000) vars and O(100k) constraints). Local methods require gradients of the constraint functions, which can only be obtained by finite differences. Values of mechanical strength constraints are computed using dedicated software. A call to this software takes up to a second; as a consequence the need for finite difference calculations in each of numerous local optimizations greatly increases the time between two update steps.

Therefore, the dimension estimation step in an aircraft development program is a repetitive, time-consuming process. Much time could be saved by using a surrogate modeling instead of performing straightforward computing [1], [2]. Thus the main motivation of this work is a surrogate modeling of buckling analysis in support of composite structure optimization. In fact, we want to achieve two goals being of great importance to engineers working in Airbus Structural Analysis Framework, namely, save time in pre-sizing processes and get the advantage of response smoothing. Indeed Surrogate Models (SMs) give a continuous and differentiable approximation of RFs that sometimes are not themselves continuous (as often for semi-empirical approaches).

For surrogate modeling of static instability phenomena (the buckling and the collapse of a super stiffener) we used MACROS software toolkit for surrogate modelling and optimization, developed by DATADVANCE [3].

Finally constructed MACROS surrogate model (MACROS SM) was embedded into pre-sizing optimization process of A350XWB composite boxes, realized in a pre-sizing tool COMBOX, for checking the validity of the approximation and its use as a constraints in an optimization process. It turned out that MACROS SM allows obtaining smoother convergence to a reasonable solution in less iterations with a smoother distribution of thickness/stringer dimensions and provides reduction of structure optimization computational time from several days to a few hours.

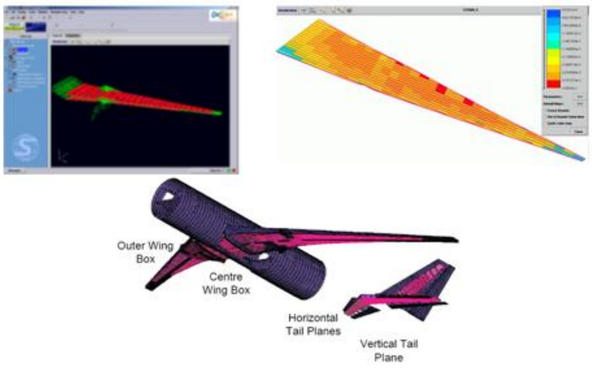


Figure 1. COMBOX pre-sizing optimization tool is now applied to all A350XWB boxes

In the following sections we outline description of the pre-sizing tool COMBOX (section 1), surrogate modelling and optimization software toolkit MACROS (section 2), construction of the

MACROS SM for skill tool (section 3), analysis of optimization results based on skill tool and constructed SM (section 4). Lastly, we end this article with some concluding remarks (section 5).

2 COMBOX: a pre-sizing tool developed for A350XWB

The COMBOX tool (COMposite BOX pre-sizing) was developed in 2005 to support the pre-sizing of the A350XWB composite wing box (see figure 1). It has then been continuously improved and is being applied to all A350XWB boxes: wing, horizontal tail plane and vertical tail plane.

2.1 COMBOX sizing process

COMBOX sizing process encapsulates the full stress process for a wing box (see figure 2):

- Mapping of sizing properties
- Update of global finite element model
- Calculation of internal loads through a static linear analysis based on the global FEM
- Calculation of strength responses as Reserve Factors (RFs) through AIRBUS skill tools

These several steps are the usual steps for an airframe structural analysis for pre-sizing.

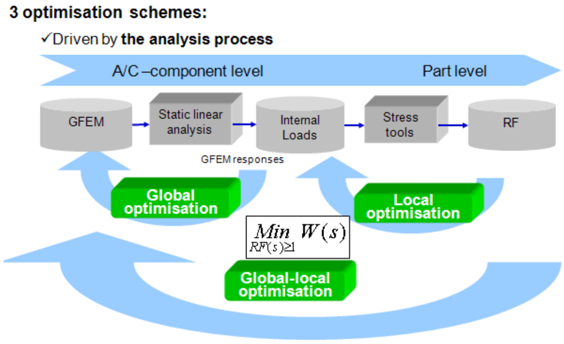


Figure 2. COMBOX pre-sizing optimization tool is a global-local optimization capability encapsulating the overall stress analysis process

Remarks:

- A Reserve Factor indicates whether the structure is feasible (that is to say has enough strength) with respect to a given mechanical criterion or failure mode. If the RF is greater than one, the structure is feasible. If the RF is less than one, it is not feasible. Therefore, when modeling dependency of an RF on a vector of design variables $\mathbf{X}$, the highest possible accuracy should be provided for so-called accuracy domain $A_e = \{\mathbf{X} : 1-e \le RF(\mathbf{X}) \le 1+e\}$ with some $0<e<1$.
- The simplest example of an RF is a ratio between a stress allowable (for example material strength) and the applied stress.
- Skill tools are usually analytical semi-empirical tools which are rather quick and used for pre-sizing.

2.2 COMBOX components

COMBOX is based on commercial off-the shelf software and incorporates four components (see figure 3):

- CAESAM: Software framework from SAMTECH
- NASTRAN: Finite Element software from MSC
- Skill tools developed by AIRBUS
- BOSS Quattro: Optimization software from SAMTECH

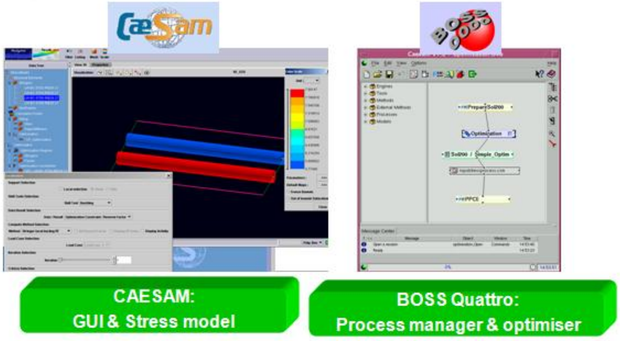


Figure 3. CAESAM/BOSS Quattro as main components of COMBOX

2.3 COMBOX optimization process

COMBOX is a pre-sizing tool based on numerical optimization (so-called mathematical programming). Therefore on top of calculations in the above process, it is necessary to compute sensitivities of internal loads and Reserve Factors and to combine them by chain ruling (see figure 4). Internal load sensitivities are semi-analytically calculated thanks to the use of NASTRAN SOL200 module (NASTRAN optimization, sensitivity analysis module). Then responses and sensitivities are sent to the optimization algorithm in BOSS Quattro. CAESAM is mainly used to manage all data and BOSS Quattro manages the workflow and the optimization process including the sensitivity chain-ruling.

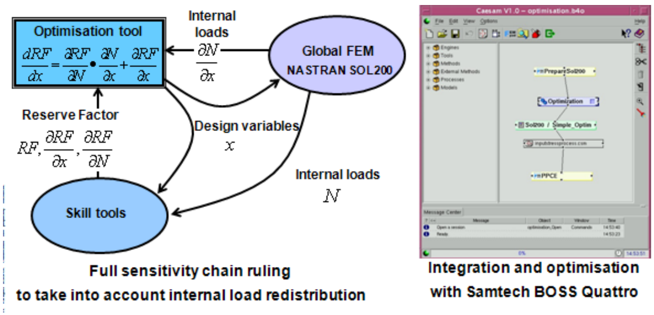


Figure 4. COMBOX optimization process

2.4 COMBOX optimization problem formulation

COMBOX is able to address all sizing variables of a composite cover with T-stringers (other stringer sections are possible but not presented here, see also figure 5):

- Skin thickness
- Percentages of standard draping angles: 0%, 45%, 90%
- T-stringer core and web percentages: 0%, 45%, 90%
- T-stringer web thickness, core thickness, height and width

Bounds are given to these various variables to satisfy design rules. Some additional design rules are included like bounds on As/bt ratio which represents the ratio of the stringer area to the skin area.
All usual criteria for a composite wing cover sizing are considered (see figure 6):

- Skin local buckling and skin general buckling
- Post-buckling and post-buckling cut-off
- Skin damage tolerance and stringer damage tolerance
- Skin reparability and stringer reparability

RFs are associated to each of these failure mode.

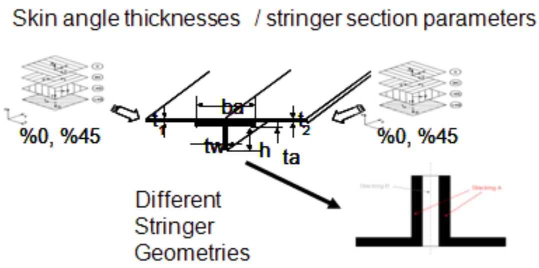


Figure 5. Illustration of COMBOX design variables

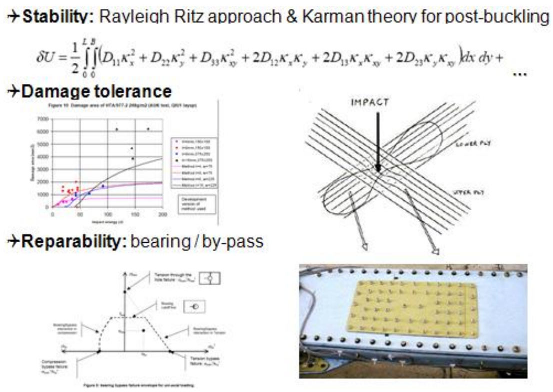


Figure 6. Illustration of COMBOX strength criteria

Damage tolerance criteria are there to ensure the structure can resist small damages. Reparability criteria anticipates some future repairs in the skin (filled hole criteria).

Therefore the optimization problem can be formulated as:

$$M(x) \to \min_{x \in \Re^n} \quad s.t. \begin{cases} x_{low} \le x \le x_{up} \\ RF_{i,j,k}(N(x), x) \ge 1,\ i = 1, N_e;\ j = 1, N_l; k = 1, N_{fm} \\ d_l\left(x\right) \ge 1 \end{cases}$$

where

- the objective function is the mass *M(x)* of the finite element model, independent from percentages, and to be minimized,
- x is the vector of the n optimization variables (skin, stringer thicknesses, dimensions and percentages),
- $N(x)$ is the vector of internal loads.

The constraints are

- Variable bounds: $x_{low} \le x \le x_{up}$
- Strength constraints: $RF_{i,j;k}(N(x), x) \ge 1$
- Design constraints: $d_l\left(x\right) \ge 1$

The index i,j,k for the strength constraints remind that there are as many strength constraints as structural elements: N_e, external loads N_l and failure modes N_{fm}. The computational time of the process is mainly contained in the strength analysis due to high value of $N_e \cdot N_l \cdot N_{fm}$. On top of that RF sensitivities being performed via finite differences, the number of strength analyses is multiplied by the number of local variables and internal load components (approximately a factor 10).

Therefore even if strength analysis tools for pre-sizing are rather quick (1s) per element, when we multiply by the total number of calculations it leads to 1,5 days per iteration with a process usually converging in 20 iterations (see figure 7 for details). To save time in pre-sizing processes and particularly in the COMBOX tool it is then necessary to built numerical approximation of strength tools using surrogate modeling, which is the main goal of this paper. On top of time reduction is also the advantage of response smoothing. Indeed SMs give a continuous and differentiable approximation of RFs that sometimes are not themselves continuous (as often for semi-empirical approaches). This was also demonstrated in the current study.

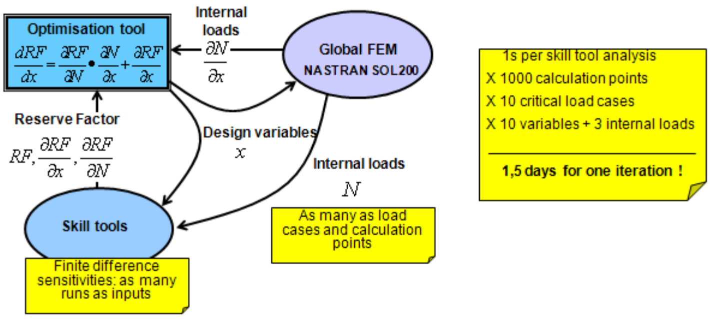


Figure 7. Computational times in COMBOX

3 MACROS: a surrogate modelling and optimization software toolkit

MACROS is a software toolkit for

- intellectual data analysis and
- multi-disciplinary optimization,

developed by DATADVANCE [3]. It provides proprietary and state-of-the-art data analysis and optimization techniques.

MACROS toolkit consists of Generic Tools (GTs) for Dimension reduction, Important Variable Extraction, Design of Experiments, Approximation, Data Fusion, Optimization.

GT for Dimension reduction includes unsupervised and supervised (so-called feature extraction) techniques for automatic re-parameterization of an object's description with a smaller number of parameters.

GT for Important Variable Extraction includes techniques for sensitivity analysis necessary for ranking the available parameters with respect to their influence on the given response function and selecting the most important ones.

GT for Design of Experiments enables systematic and efficient analysis of the design space by using classical and advanced methods (full factorial, optimal Latin Hypercube, Halton and Sobol sequences, etc.) as well as specially designed adaptive techniques.

GT for Approximation allows automatic construction of fast-running data based SMs using best-in-class predictive modeling techniques. The tool includes built-in robustness and accuracy assessment, control of SM smoothness, etc. and is efficient for small and huge data samples in low and high dimensions.

GT for Data Fusion allows approximating data of variable fidelities. The tool operates like GT for Approximation, but assumes that the response function is represented by two types of data: scarce high fidelity data and abundant low fidelity data. The tool then constructs an enhanced approximation of the high fidelity model taking into account the abundant low fidelity data.

GT for Optimization includes efficient state-of-the-art optimization methods to solve various problems (large scale, linear/nonlinear, unconstrained/constrained, single/multi-objective and stochastic).

Adaptive and automatic selection of the best method for a given problem on basis of specially designed decision trees opens up elaborated methods for use by people who interact with problems on the engineering rather than mathematical level.

4 Construction of MACROS SM

Mathematically the process of construction of SM $\hat{y} = F_{surr}(\mathbf{X})$ for some unknown function $y = F(\mathbf{X})$ consists of several steps, realized using MACROS toolkit:

- Automatic generation of the training sample $S = \{y_i, \mathbf{X}_i\}_{i=1}^{N}$, where y_i is the value of RF for the corresponding input vector (in the considered case $\mathbf{X}_i \in \mathbb{R}^{20}$, $N \sim 200000$).
- Let $y_{\min}$ and $y_{\max}$ be the upper and lower bounds of the output value $y = F(\mathbf{X})$. The interval of variation $y \in [y_{\min}, y_{\max}]$ is divided into K subintervals, i.s. $[y_{\min}, y_{\max}] = \bigcup_{j=1}^{K} [y^{j-1}, y^{j}]$, where $y^0 = y_{\min}$, $y^K = y_{\max}$. In general case the subintervals can intersect. The selection of particular decomposition is done either automatically or by human expert on basis of the physical sense of approximated dependency. For example, when approximating RF the accuracy domain $A_e = \{\mathbf{X} : 1 - e \le F(\mathbf{X}) \le 1 + e\}$, $e = 0.2$ was included into such decomposition. This allows to significantly increase the accuracy of approximation in this domain, which is important for the considered application.
- The sample S is divided into K subsamples $S = \bigcup_{j=1}^{K} S_j$ such that $S_j = S \cap [y^{j-1}, y^{j}]$.
- On basis of each subsample S_j an approximator $F^{j}_{approx}(\mathbf{X})$ is constructed. For its construction preliminary clustering can also be used.
- On basis of the sample S the classifier $F_{class}(\mathbf{X})$ is constructed. The classifier for each input $\mathbf{X}$ returns the subset of the constructed approximators $F^{j}_{approx}(\mathbf{X})$, $j \in J(\mathbf{X}) \subseteq \{1, \dots, K\}$, which should be used to predict the corresponding RF value y.

Two steps are done in order to calculate the prediction $\hat{y} = F_{surr}(\mathbf{X})$:

- Using the classifier $F_{class}(\mathbf{X})$ the corresponding subset $F^{j}_{approx}(\mathbf{X})$, $j \in J(\mathbf{X}) \subseteq \{1, \dots, K\}$ of the constructed approximators are chosen.
- Calculate the prediction $F_{surr}(\mathbf{X}) = \sum_{j \in J(\mathbf{X})} \omega(j, \mathbf{X}) \cdot F^{j}_{approx}(\mathbf{X})$, where the weights $\omega(j, \mathbf{X})$, $j \in J(\mathbf{X}) \subseteq \{1, \dots, K\}$ are such that $\sum_{j \in J(\mathbf{X})} \omega(j, \mathbf{X}) = 1$ and $\omega(j, \mathbf{X}) > 0$.

Experiments [4] showed that MACROS SMs have significantly higher accuracy compared to conventional methods (Kriging, Artificial Neural Networks, Support Vector Regression, Multivariate Nonparametric Regression, Polynomial Regression, etc.]) and specially designed method for the considered problem [5]. Particular MACROS algorithms for construction of approximators $F^{j}_{approx}(\mathbf{X})$, $j = 1, 2, \dots, K$ etc. are discussed, for example, in [5], [6].

5 Results of optimization based on skill tool and MACROS SM

In this section some results of using SMs in pre-sizing optimization tool COMBOX for composite structures are presented. The classical skill tool called PS3 (Plane Super-Stiffener Sizing) was replaced by MACROS SM. The objective of this study was to check the impact of this replacement on the accuracy, on the convergence of the optimization process and on the run time.

5.1 Optimization runs

In this study the optimization was performed on the wing lower and upper covers as described in figure 8. Two test cases for the optimization study were considered corresponding to 2 starting points:

- The first one is close to an optimal design, obtained with using only PS3 skill tool.
- The second one is a heavy one, where all design variables are set to their upper bound. This last run illustrates the behaviour using SMs for a complete optimization run.

For each test case first run was done with MACROS SM (until convergence). Then update and restart with PS3 was performed. Due to limited space only some representative results are shown.

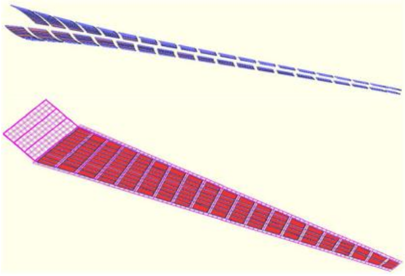


Figure 8. A30X WING Stress Model

5.1.1 Optimization runs: initial starting point

Comparison with "pure" PS3 run is presented in figure 9 (blue-green curve corresponds to MACROS SM and subsequent update with PS3; orange curve corresponds to a run with only PS3).

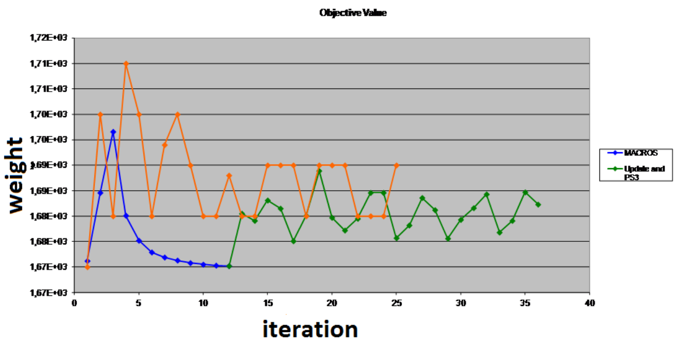


Figure 9. Evolution of the objective function for initial starting point

We can observe more chaotic evolution with PS3 due to the proximity to the optimum solution. But also linked to some discontinuities in the Reserve Factors calculated with PS3.

5.1.2 Optimization runs: heavy starting point

Evolution of the objective value is given in figure 10 (blue curve corresponds to MACROS SM, green curve corresponds to restart from MACROS SM with PS3, orange curve corresponds to PS3 only). We can observe smooth evolution with both MACROS SM (SM is a continuous function) and PS3 (the behavior of PS3 is smoother than in the previous run, since the starting point is not so close to the optimal design) but the evolution is nevertheless smoother with MACROS SM.

Evolution of the numbers of violated constraints (left) and saturated constraints (right) is given in figure 11. Peak at the beginning of the left plot appears due to strategy of active constraints. We can observe increase of the number of violated constrainst when updating the model (due to the switch of skill tool) and after a quick decrease of it. Decrease of the number of saturated constraints at iteration six happened due to the re-actualization of the active constraints and the identification of new violated constraints, decrease of the number of saturated constraints from MACROS SM to restart with PS3 happened due to the switch of the skill tool. The restart with PS3 presents less saturated constraints than MACROS SM. This proves a better quality of the optimum found with MACROS SM, which is linked to the smoothness and mathematical differentiability of MACROS SM in contrast to PS3.

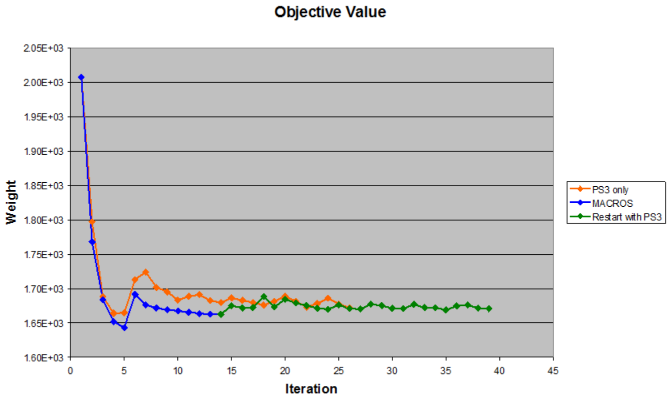


Figure 10. Evolution of the objective function for heavy starting point

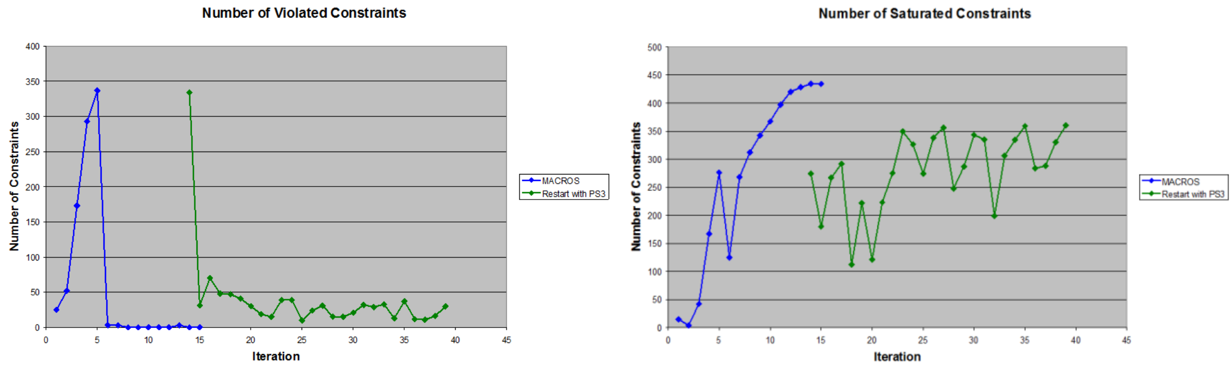


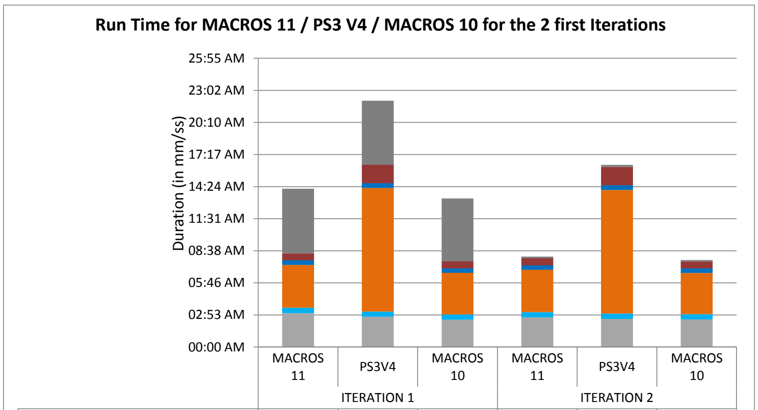
Figure 11. Evolution of the numbers of violated (left) and saturated (right) constraints

5.2 Execution time: comparison between PS3 skill tool and MACROS SM

The objective of this study was to compare the run time of MACROS SM and PS3 skill tool for one iteration of optimization process. The study was done on the test case 1 (initial point) and on the 2 first iterations. The sequence of computations for 1 iteration is the following:

- NASTRAN SOL200: for internal loads update and sensitivities analysis,
- Computation of the RFs corresponding to the current design with the PS3 stress process or the MACROS SM,
- Computation of the Interregional Constraints corresponding to the current design,
- Computation by using PS3 or MACROS of the sensitivities for all the RFs,
- Computation of the sensitivities for the Interregional Constraints.

We observed an overall gain factor of at least 2.5 by using the MACROS SM instead of PS3 (thanks to acceleration of the steps “MACROS or PS3 Nominal” and “MACROS or PS3 Sensitivities”, see figure 12). Since MACROS SM provides very fast analytical computation of sensitivities, then further significant speed up can be obtained by using analytical sensitivity computations instead of numerical. The speed-up factor is less than expected because there is a large system time spent in the management of jobs via LSF.



	MACROS 11	PS3V4	MACROS 10	MACROS 11	PS3V4	MACROS 10
	ITERATION 1			ITERATION 2		
NASTRAN SOL 200	00:05:49	00:05:45	00:05:38	00:00:09	00:00:11	00:00:08
MACROS or PS3 Nominal	00:00:37	00:01:40	00:00:38	00:00:38	00:01:39	00:00:37
Inter-regional Constraints Nominal	00:00:25	00:00:25	00:00:24	00:00:24	00:00:25	00:00:25
MACROS or PS3 Sensitivities	00:03:50	00:11:06	00:03:44	00:03:48	00:11:06	00:03:42
Inter-regional Constraints Sensitivities	00:00:29	00:00:28	00:00:29	00:00:28	00:00:28	00:00:28
Additional Time	00:03:03	00:02:43	00:02:27	00:02:40	00:02:32	00:02:29

Figure 12. Run Time (hh/mm/ss) for MACROS 11 / PS3 V4 / MACROS 10 on Iterations 1 and 2

5.3 Check of reserve factors

A check of reserve factors was performed with the optimum based on MACROS SM. Results are presented in the figure 13. These results show a satisfactory accuracy for a pre-sizing result, according to AIRBUS experts and considering that a pre-sizing is always to be re-engineered including for example manufacturing constraints. Work is on-going to further improve the accuracy.

	RF EHV	Rib Bay																			
	Stringer	1-2	2-3	3-4	4-5	5-6	6-7	7-8	8-9	9-10	10-11	11-12	12-13	13-14	14-15	15-16	16-17	17-18	18-19	19-20	20-21
Top Cover	1	1.01																			
	2	1.03	0.95	1.00	0.99	0.98															
	3	0.99	0.99	0.99	0.97	0.98	1.00	0.99	0.95												
	4	1.03	1.02	1.01	1.00	0.99	1.01	1.00	0.96	0.97	1.04	0.98	0.97								
	5	0.99	0.97	0.97	0.98	0.97	0.97	0.97	0.96	0.99	0.95	0.95	0.95	1.01	0.98	0.95	0.99	0.95			
	6	0.99	0.95	0.96	0.97	0.96	0.96	0.99	0.97	1.00	0.95	0.96	0.95	1.04	0.97	0.95	0.98	0.96	1.00	0.91	1.01
	7	0.99	0.96	0.96	0.96	0.95	0.95	0.97	0.97	0.98	1.05	0.95	0.96	0.93	0.95	0.92	0.98	0.67			
	8	0.99	0.97	0.95	0.95	0.94	0.95	0.96	0.97	0.99	0.98	0.93	0.96	0.90							
	9	1.01	0.94	0.98	0.96	0.94	0.98	0.96	0.92	0.93											
	10	0.99	0.99	1.01	1.01	0.94															
	11	1.02																			

Figure 13. Check with PS3 of optimum results found with MACROS SM (wing top cover)

6 Conclusions

Constructed MACROS SMs were embedded into **pre-sizing optimization process of A350XWB boxes** realized in pre-sizing tool COMBOX for checking the validity of the approximation and its use as a constraints in an optimization process. The analysis/comparison of optimization results based on skill tool and optimization results based on constructed MACROS SM was performed and showed that

- MACROS surrogate model gives high accuracy of approximation (see also [5])
- MACROS surrogate model allows obtaining smoother convergence in less iterations with a smoother distribution of thickness/stringer dimensions and a small violation of constraints which then could be easily repaired at the detailed design phase
- MACROS surrogate model provides reduction of structure optimization computational time **from several days to a few hours**.

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